Perspectives on Nodal Superconductors

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Abstract

In the last few years the gap symmetries of many new superconductors,including Sr_2RuO_4, $CeCoIn_5$, κ-$(ET)_2Cu(NCS)_2$, YNi_2B_2C and $PrOs_4Sb_{12}$, have been identified via angle-dependent magnetothermal conductivity measurements. However, a controversy still persists as to the nature of the superconductivity in Sr_2RuO_4. For $PrOs_4Sb_{12}$, spin-triplet superconductivity has recently been proposed. Here, we also propose g-wave superconductivity for UPd_2Al_3 (i.e., $\Delta(\mathbf{k}) = \Delta\cos(2\chi), \chi = ck_z$) based on recent thermal conductivity data.

PACS numbers:

1. Introduction

After the appearance of heavy-fermion superconductors and organic superconductors in 1979 the gap symmetries of these new compounds have been a central issue[1]. However, until recently only the $d_{x^2-y^2}$-wave symmetry of the gap function $\Delta(\mathbf{k})$ in high-T_c cuprates has been established by the elegant Josephson interferometry[2] and the angle resolved photoemission spectra (ARPES)[3]. Unfortunately, so far these powerful techniques are unavailable for heavy-fermion superconductors and organic superconductors with lower superconducting transition temperatures $T_c \leq 10K$.

In the last few years, Izawa et al have established the gap symmetries of superconductivity in Sr_2RuO_4[4],$CeCoIn_5$[5], κ-$ET_2Cu(NCS)_2$[6], YNi_2B_2C[7] and $PrOs_4Sb_{12}$[8] through the angle dependent magnetothermal conductivity. This breakthrough relies in part on the availability of high-quality single crystals of these compounds and in part on the theoretical development initiated by Volovik.[9] Last year, we have reviewed the progress in [10].

In the present paper, we focus on 3 recent topics in nodal superconductors. In spite of ample evidence for f-wave superconductivity in Sr_2RuO_4 [10] the controversy regarding this compound appears to continue. Therefore in section 2 we discuss the angle dependent magnetospecific heat data by Deguchi et al [11]. Now evidence for spin-triplet superconductivity in $PrOs_4Sb_{12}$ is mounting. In section 3, we describe p+h-wave superconductivity for the A and B phases in $PrOs_4Sb_{12}$[12]. Recently angle-dependent thermal conductivity data in the vortex state in UPd_2Al_3 has been reported.[13] In section 4 we analyze the angle-dependent magnetothermal conductivity κ_{yy} when the field is rotated within the z-x plane, and we conclude that $\Delta(\mathbf{k})$ in UPd_2Al_3 is given by $\Delta(\mathbf{k}) = \Delta\cos(2\chi)$[14]. In Fig. 1 we show the new $|\Delta(\mathbf{k})|'s$ so far identified.

2. F-wave Superconductivity in Sr_2RuO_4

Superconductivity in Sr_2RuO_4 was discovered in 1994[15]. Sr_2RuO_4 is an isocrystal to La_2CuO_4, but it is metallic down to low temperatures and becomes superconducting around T = 1.5 K. An early review on Sr_2RuO_4 can be found in Ref.[16]. From the analogy to superfluid ^{3}He Rice and Sigrist[17] proposed 2D p-wave superconductivity. Indeed spin-triplet pairing and related chiral symmetry-breaking have been established [18, 19, 20]. As sample quality improved around 1999, both the specific heat data [21] and the superfluid density [22] indicated nodal structure in the superconducting order parameter of Sr_2RuO_4. These findings ruled out p-wave superconductivity and its generalization[23]. Therefore, a variety

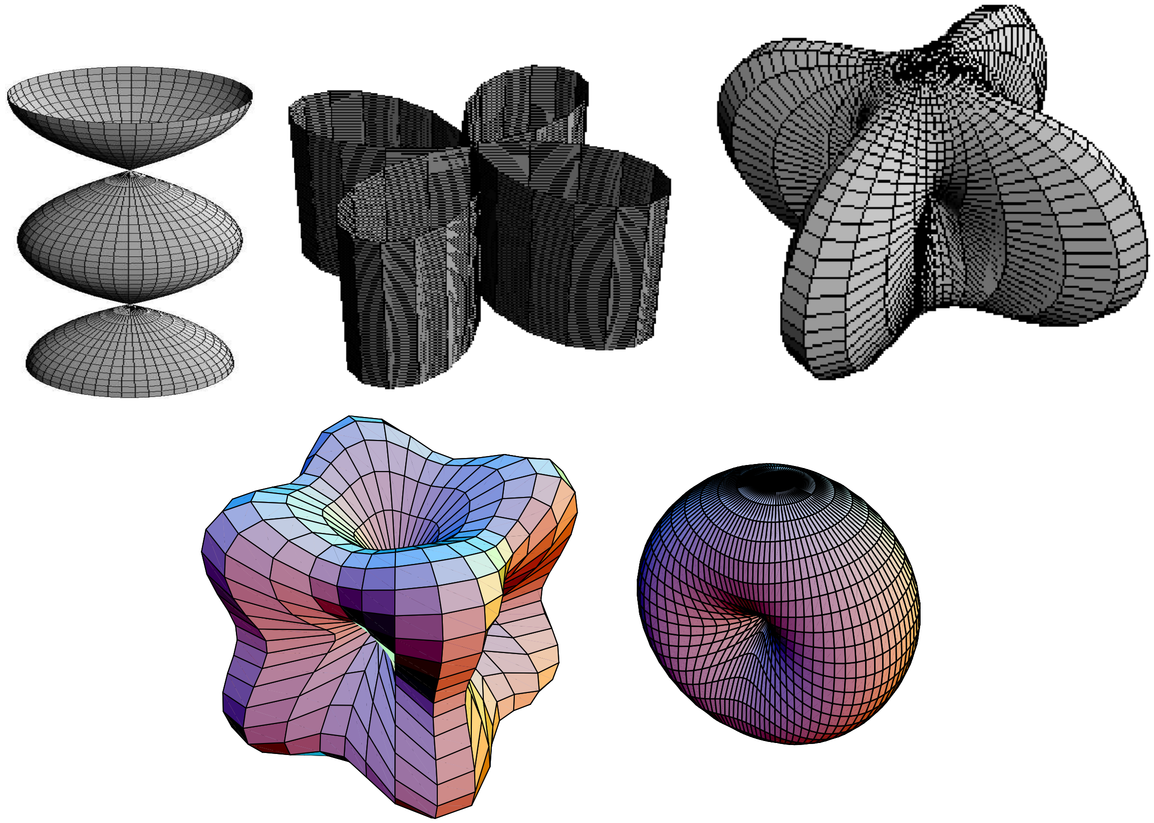

FIG. 1: From top left, 2D f-wave - Sr_2RuO_4, $d_{x^2-y^2}$-wave - $CeCoIn_5$ and κ-$(ET)_2Cu(NCS)_2$,s+g-wave - YNi_2B_2C, p+h-wave - $PrOs_4Sb_{12}$ - A phase, p+h-wave - $PrOs_4Sb_{12}$ - B phase.

of f-wave order parameters were suggested. [24] In Fig.2 and Fig.3 we show the specific heat data [21] and the superfluid density data[22] compared with a variety of models.

However, these experiments cannot tell us about the nodal structure of $\Delta(\mathbf{k})$. In a quasi-2D system such as Sr_2RuO_4, the line nodes in $\Delta(\mathbf{k})$ can be either vertical or horizontal. But vertical nodes are incompatible with the angular dependent magnetothermal conductivity [4] and the ultrasonic attenuation data [25]. Furthermore, Ref. 4 indicates that the horizontal nodes are far away from χ_0=0. This suggests $\Delta(\mathbf{k}) = \mathbf{d}e^{\pm i\phi}\cos(\chi)$, i.e. 2D f-wave superconductivity[26].

This interpretation is contested by Deguchi et al [11]. They measured the magnetospecific heat of Sr_2RuO_4 in a rotating magnetic field down to 100 mK and found cusp-like features only in the regime $0.12K < T < 0.31K$. From our earlier analysis of s+g-wave superconductivity[27, 28], we deduce that there should be a point-like minigap with

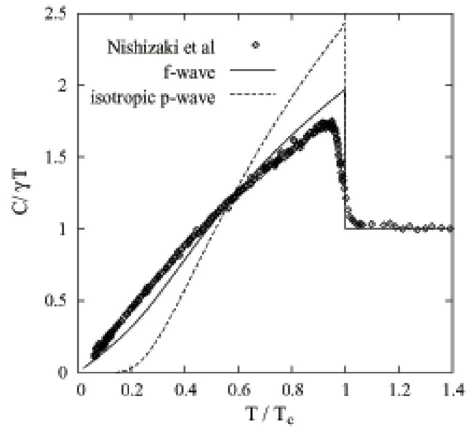

FIG. 2: Specific heat for 2D p-wave and f-wave models for Sr_2RuO_4.

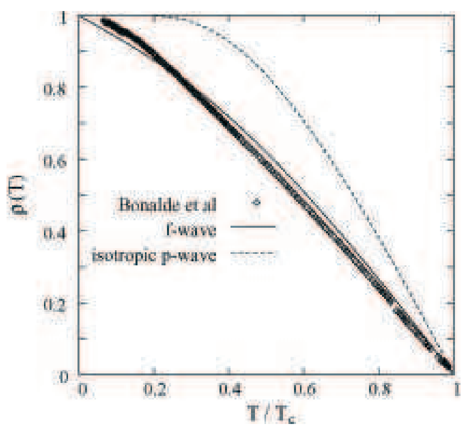

FIG. 3: Superfluid density for 2D p-wave and f-wave models for Sr_2RuO_4.

$\Delta_{min} \sim 0.1K$. The simplest triplet gap function which has these minigaps is

$$\Delta(\mathbf{k}) = \mathbf{d}e^{\pm i\phi}(1 + a\cos(4\phi)\cos(\chi)) \tag{1}$$

where $|1 - a| \leq 0.1$. Deguchi et al have proposed the Miyake-Nariyiko (MN) model [29], in order to describe the measured specific heat. However, it is easy to see that the MN model cannot give the cusp-like features in the magnetospecific heat. Also, the MN model cannot describe the observed T^2 specific heat or the T-linear dependence of the superfluid density. Moreover, the angular dependent thermal conductivity data and the universal heat conduction in κ_{xx} by Suzuki et al [30] are incompatible with the MN model. Therefore further experiments on Sr_2RuO_4 are highly desirable. We have proposed that the optical conductivity [31], the Raman scattering[32] and the supercurrent [33, 34] in Sr_2RuO_4 will provide further insight on its superconductivity.

3. Triplet Superconductivity in $PrOs_4Sb_{12}$

Superconductivity with $T_c = 1.8$ K has been discovered very recently in the skutterudite $PrOs_4Sb_{12}$[35, 36, 37]. Angle-dependent thermal conductivity data on this system has revealed a multi-phase structure, characterized by a gap function with point nodes.[8] In order to account for this nodal structure s+g-wave superconductivity has been proposed. [10, 38]

Recently there has been mounting experimental evidence for triplet superconductivity in this compound. First, from μSR measurements Aoki et al discovered a remnant magnetization in the B-phase of this compound, indicating triplet pairing. [39] Second, the thermal conductivity measurement in a magnetic field down to low-temperature ($T > 150$ mk) indicates $\kappa_{zz} \sim T$ and H [40], consistent with triplet pairing. Later we shall discuss κ_{zz} measured in a magnetic field rotated within the z-x plane. This data is fully consistent with triplet p+h-wave superconductivity in $PrOs_4Sb_{12}$. Finally, a recently reported NMR result for the Knight shift by Tou et al [41] also suggests the triplet pairing. Here we propose p+h-wave order parameters

$$\mathbf{\Delta}_A(\mathbf{k}) = \frac{3}{2}\mathbf{d}e^{\pm i\phi_1 \pm i\phi_2 \pm i\phi_3}(1 - \hat{k}_1^4 - \hat{k}_2^4 - \hat{k}_3^4)) \quad (2)$$

$$\mathbf{\Delta}_B(\mathbf{k}) = \mathbf{d}e^{\pm i\phi_3}(1 - \hat{k}_3^4) \quad (3)$$

for the A-phase and B-phase of $PrOs_4Sb_{12}$, respectively, where $e^{\pm i\phi_1} = \hat{k_2} \pm i\hat{k_3}$, etc. These order parameters have nodal structures consistent with the angle dependent thermal conductivity data [8], assuming that in the experiment the nodes in the B-phase are aligned parallel to the y-axis. We note $|\Delta(\mathbf{k})|$ in the A-phase has cubic symmetry whereas in the B-phase it has axial symmetry. Furthermore, it appears that in the slow field-cooled situation the nodes in the B-phase are aligned parallel to the magnetic field. At least this is the simplest way to interpret the superfluid density measurement by Chia et al [42, 43].

Here we give expressions for the thermal conductivity κ_{zz} in a magnetic field in the superclean limit ($\sqrt{\Gamma\Delta} \ll v\sqrt{eH}$),

$$\kappa_{zz}/\kappa_n = \frac{v^2 eH}{8\Delta^2}\sin^2(\theta) \quad , A - phase \quad (4)$$

$$= \frac{3v^2 eH}{64\Delta^2}\sin^2(\theta) \quad , B - phase \quad (5)$$

and in the clean limit ($v\sqrt{eH} \ll \sqrt{\Gamma\Delta}$)

$$\kappa_{zz}/\kappa_{00} = 1 + \frac{3v^2eH}{40\Gamma\Delta}\ln(\sqrt{\frac{2\Delta}{\Gamma}})\sin^2(\theta)\ln(\frac{\Delta}{v\sqrt{eH}\sin(\theta)}), \quad A-phase \tag{6}$$

$$= 1 + \frac{v^2eH}{12\Gamma\Delta}\ln(\sqrt{\frac{2\Delta}{\Gamma}})\sin^2(\theta)\ln(\frac{\Delta}{v\sqrt{eH}\sin(\theta)}), \quad B-phase \tag{7}$$

where κ_n and κ_{00} are the thermal conductivity in the normal state and the thermal conductivity in the limit of universal heat conduction $\Gamma \to 0, T \to 0$. Here, Γ is the quasiparticle scattering rate in the normal state, and θ is the angle $\mathbf{H}$ makes from the $\hat{z}$ axis. In both Eq.(5) and Eq.(7) we have assumed that the nodes in the B-phase are parallel to the z axis. Otherwise κ_{zz} is smaller by a factor of $10 \sim 50$. In Fig. 4 we compare the observed angle dependent thermal conductivity with Eq.(6) and (7). These equations give an excellent fit.

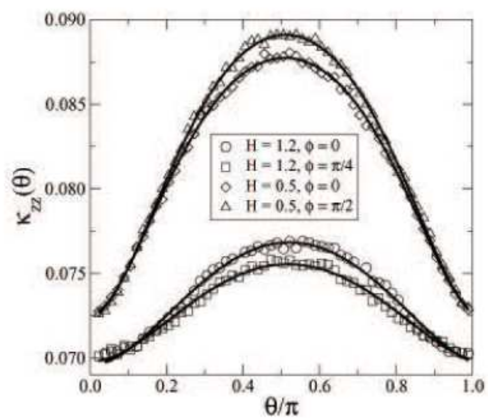

FIG. 4: Angular-dependent thermal conductivity in $PrOs_4Sb_{12}$.

From this we extract $v = 0.96 \times 10^7$ cm/sec and $\Gamma = 0.1$ K, where use is made of the weak-coupling theory gaps $\Delta_A = 4.2K$ and $\Delta_B = 3.5K$ for the A and B phase respectively. Note that de Haas-van Alphen measurements [44] give comparable values of v (0.7×10^7 cm/sec [α-band], 0.6×10^7 cm/sec [β - band] and 0.23×10^7 cm/sec [γ band]).

4. G-wave superconductivity in UPd_2Al_3

This heavy-fermion superconductor with $T_c \simeq 2K$ was discovered by Geibel et al [45] in 1991. The reduction of the Knight shift in the superconducting state seen in NMR [46] and the Pauli limiting of H_{c2} in UPd_2Al_3[47] established spin- singlet pairing. Nodal superconductivity with horizontal nodes has been deduced from the thermal conductivity

[48] and the c-axis tunneling data from UPd_2Al_3 thin films [49]. Here we focus on thermal conductivity data reported in Ref.[13]. First, κ_{zz} at T = 0.4 K in a rotating magnetic field was measured. For H < 0.5 T no ϕ dependence was seen, (ϕ is the angle **H** makes from the x axis). This indicates that the nodal lines should be horizontal. Second, κ_{yy} in a magnetic field rotated within the z-x plane was measured.

We show this in Fig. 5. Following the standard procedure [10], the thermal conductivity

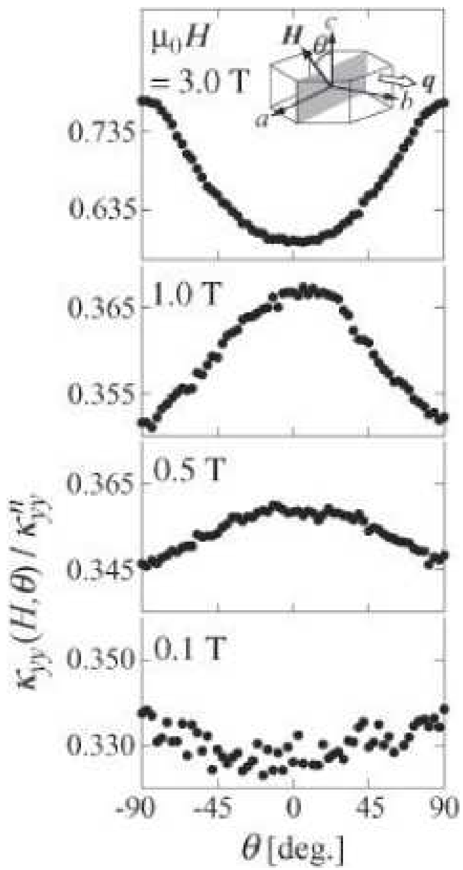

FIG. 5: Angular dependent magnetothermal conductivity in UPd_2Al_3

κ_{yy} is obtained for a variety of $\Delta(\mathbf{k})$ with horizontal nodes as

$$\kappa_{yy}/\kappa_n = \frac{2}{\pi}\frac{v_a^2 eH}{\Delta^2}F_1(\theta) \tag{8}$$

in the superclean limit and

$$\kappa_{yy}/\kappa_{00} = 1 + \frac{v_a^2 eH}{6\pi\Gamma\Delta}F_2(\theta)\ln(2\sqrt{\frac{2\Delta}{\pi\Gamma}})\ln(\frac{\Delta}{v\sqrt{eH}}) \tag{9}$$

in the clean limit, where

$$F_1(\theta) = \sqrt{\cos^2\theta+\alpha\sin^2\theta}(1+\sin^2\theta(-\frac{3}{8}+\alpha\sin^2\chi_0)) \tag{10}$$

$$F_2(\theta) = \sqrt{\cos^2\theta+\alpha\sin^2\theta}(1+\sin^2\theta(-\frac{1}{4}+\alpha\sin^2\chi_0)), \tag{11}$$

and $\alpha=(v_c/v_a)^2$. θ is the angle $\mathbf{H}$ makes from the z-axis and χ_0 is the nodal position. For $\Delta(\mathbf{k})\sim\cos\chi, \cos(2\chi)$, and $\sin\chi$ we obtain $\chi_0=\frac{\pi}{2},\frac{\pi}{4}$ and 0, respectively. We show $F_1(\theta)$ and $F_2(\theta)$ in Fig. 6 where we used $\alpha=0.69$, the appropriate value for UPd_2Al_3. A comparison of Fig. 5 and Fig. 6 indicates that $\Delta(\mathbf{k})\sim\cos(2\chi)$ is the most appropriate choice. Similarly

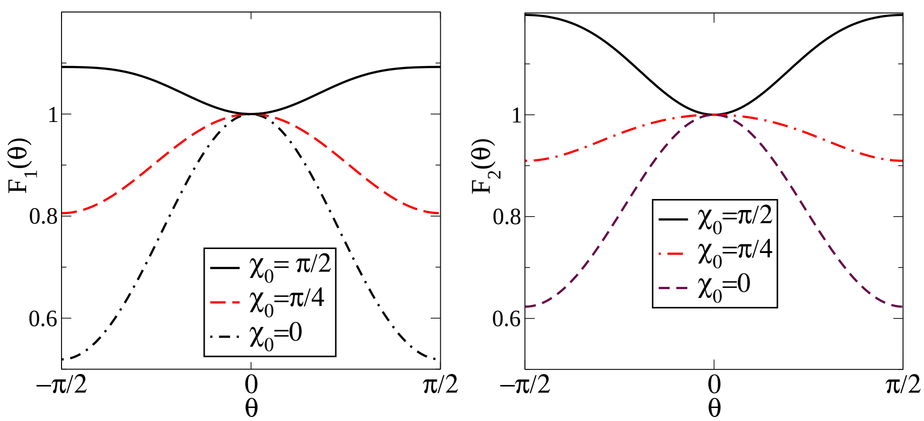

FIG. 6: Angular functions $F_1(\theta)$ (left) and $F_2(\theta)$ for various nodal positions

the universal heat conduction in nodal superconductors [50, 51] for a variety of quasi-2D systems (f=$\cos(2\phi), \sin(2\phi), \cos\chi, cos(2\chi), \sin\chi$) is a quantity of interest. We obtain [51]

$$\frac{\kappa_{xx}}{\kappa_n} = \frac{2\Gamma_0}{\pi\Delta}\frac{1}{\sqrt{1+C_0^2}}E(\frac{1}{\sqrt{1+C_0^2}}) = I_1(\frac{\Gamma}{\Gamma_0}) \tag{12}$$

for all f's given above. Here $\Gamma_0=0.866T_c$ and C_0 is determined by

$$\frac{C_0^2}{\sqrt{1+C_0^2}}K(\frac{1}{\sqrt{1+C_0^2}}) = \frac{\pi\Gamma}{2\Delta}, \tag{13}$$

and K(z) and E(z) are the complete elliptic integrals. Here κ_n is the thermal conductivity in the normal state with $\Gamma=\Gamma_0$. Eq.(12) tells us that κ_{xx} cannot discriminate between different nodal structures. On the other hand, we find

$$\frac{\kappa_{zz}}{\kappa_n} = I_1(\frac{\Gamma}{\Gamma_0}) \;\; for \;\; f=\cos(2\phi),\sin(2\phi),\cos(2\chi), \tag{14}$$

but

$$\frac{\kappa_{zz}}{\kappa_n} = \frac{4\Gamma_0}{\pi\Delta\sqrt{1+C_0^2}}(E(\frac{1}{\sqrt{1+C_0^2}}) - C_0^2(K(\frac{1}{\sqrt{1+C_0^2}}) - E(\frac{1}{\sqrt{1+C_0^2}}))) \tag{15}$$

$$= I_2(\frac{\Gamma}{\Gamma_0}) \;\; for \;\; f = \cos\chi, e^{\pm i\phi}\cos\chi \tag{16}$$

and

$$\frac{\kappa_{zz}}{\kappa_n} = \frac{4\Gamma_0 C_0^2}{\pi\Delta\sqrt{1+C_0^2}}(K(\frac{1}{\sqrt{1+C_0^2}}) - E(\frac{1}{\sqrt{1+C_0^2}})) \tag{17}$$

$$= I_3(\frac{\Gamma}{\Gamma_0}) \tag{18}$$

for f=$\sin\chi$ and $e^{\pm i\phi}\sin\chi$. We show $I_1(\Gamma/\Gamma_0), I_2(\Gamma/\Gamma_0)$ and $I_3(\Gamma/\Gamma_0)$ in Fig. 7. Watanabe et

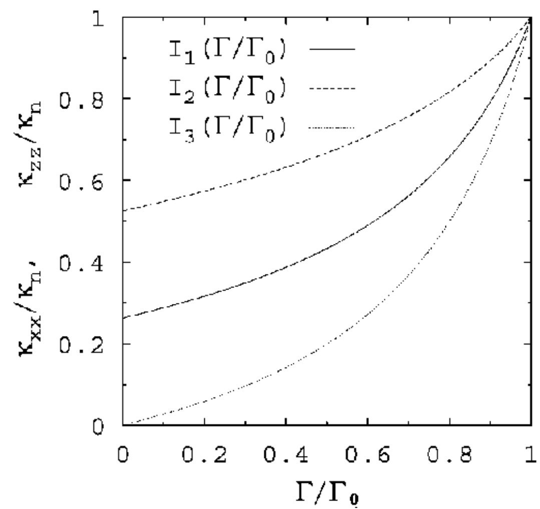

FIG. 7: The functions I_1, I_2 and I_3

al [13] also measured κ_{xx} and κ_{zz} as a function of $\mathbf{H}(||\hat{z})$. Of course the effects of impurity scattering and of magnetic fields are very different. Nevertheless the comparison of these figures suggests again $\Delta(\mathbf{k}) \sim \cos(2\chi)$.

5. Concluding Remarks

We have surveyed recent developments on nodal superconductors. As to the superconductivity in Sr_2RuO_4 we believe the 2-D f-wave model with horizontal nodes is most promising, in spite of the new specific heat data by Deguchi et al. However, further experiments on Sr_2RuO_4 are clearly desirable.

The p+h-wave superconductivity in $PrOs_4Sb_{12}$ appears to solve many jigsaw puzzles simultaneously. These superconducting order parameters are highly degenerate due to mul-

tiple chiral symmetry breaking. We expect exciting topological defects in these systems associated with the chiral symmetry breaking.

Also UPd_2Al_3 is the first Uranium compound examined through angle-dependent thermal conductivity experiments. From a more indirect way, the gap symmetry of UPt_3 has been deduced to be $f = e^{\pm 2i\phi} \sin^2\theta \cos\theta$ or E_{2u} at least for the B phase[52]. It has been shown that there are many triplet superconductors, including UPt_3, UBe_{13}, URu_2Si_2 and UNi_2Al_3 [53]. The determination of the gap symmetries of these superconductors is of great interest.

Of course the gap symmetry itself cannot tell the underlying pairing mechanism of these systems. But at least this provides the first important step for further exploration. Also, phonons most likely play no role in the pairing mechanism of most nodal superconductors. The majority of these pairings appear to be due to the antiparamagnon exchange. But we can expect more exotic interactions as well in this plethora of nodal superconductors.

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[1] M. Sigrist and K. Ueda, Rev. Mod. Phys. **63**, 239 (1991).

[2] C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. **72**, 969 (2000).

[3] A. Damascelli, Z. Hussain and Z.X. Shen, Rev. Mod. Phys. **75**, 473 (2003).

[4] K. Izawa et al, Phys. Rev. Lett. **86**, 2653 (2001).

[5] K. Izawa et al, Phys. Rev. Lett. **87**, 57002 (2001).

[6] K. Izawa et al, Phys. Rev. Lett. **88**, 27002 (2002).

[7] K. Izawa et al, Phys. Rev. Lett. **89**, 137006 (2002).

[8] K. Izawa et al, Phys. Rev. Lett. **90**, 11701 (2003).

[9] G.E. Volovik, JETP Letters **58**, 469 (1993).

[10] H. Won, Q. Yuan, P. Thalmeier and K. Maki, Brazilian J. Phys. **33**, 675 (2003).

[11] K. Deguchi, Z.Q. Mao, H. Yaguchi and Y. Maeno, Phys. Rev. Lett. **92**, 047002 (2004); K.

Deguchi, Z.Q. Mao, Y. Maeno, J. Phys. Soc. Jpn. **73**, 1313 (2004).

[12] K. Maki, S. Haas, D. Parker, H. Won, K. Izawa and Y. Matsuda, cond-mat/0406492.

[13] T. Watanabe et al, cond-mat/0405211.

[14] H. Won and K. Maki, SCES 2004 proceeding.

[15] Y. Maeno et al, Nature **372**, 532 (1994).

[16] A.P. Mackenzie and Y. Maeno, Rev. Mod. Phys. **75**, 657 (2003).

[17] T.M. Rice and M. Sigrist, J. Phys. Cond. Matt. **7**, L643 (1995).

[18] G.M. Luke et al, Nature **394**, 558 (1998).

[19] K. Ishida et al, Nature **396**, 653 (1998).

[20] R. Jin, Y. Liu, Z.Q. Mao and Y. Maeno, Europhys. Lett **51**, 341 (2000).

[21] S. Nishizaki, Y. Maeno and Z.Q. Mao, J. Phys. Soc. Jpn. **69**, 572 (2001).

[22] I. Bonalde et al, Phys. Rev. Lett. **85**, 4775 (2000).

[23] M.E. Zhitomirsky and T.M. Rice, Phys. Rev. Lett. **87**, 057001 (2001).

[24] T. Dahm, H. Won and K. Maki, cond-mat/0006307.

[25] C. Lupien et al, Phys. Rev. Lett. **86**, 5986 (2001).

[26] K. Maki and H. Won in “Fluctuating Paths and Fields”, edited by W. Jahnke et al, World Scientific, Singapore 2001.

[27] K. Maki, P. Thalmeier and H. Won, Phys. Rev. B**65**, R140502 (2002).

[28] K. Maki, H. Won and S. Haas, Phys. Rev. B**69**, 012502 (2004).

[29] K. Miyake and O. Narikiyo, Phys. Rev. Lett. **83**, 1423 (1999).

[30] M. Suzuki et al, Phys. Rev. Lett. **88**, 227004 (2002).

[31] B. Dora, K. Maki, A. Virosztek, Europhys. Lett, **62**, 426 (2003).

[32] H-Y. Kee, K. Maki, C.H. Chung, Phys. Rev. B**67**, 186504(R) (2003).

[33] H-Y. Kee, Y.B. Kim and K. Maki, Phys. Rev. B (in press).

[34] I. Khavkine, H-Y. Kee and K. Maki, cond-mat/0405236.

[35] E.D. Bauer, N.A. Frederick, P-C. Ho, V.S. Zapf and M.B Maple, Phys. Rev B**65**, R100506 (2002).

[36] R. Vollmer, M. Etzkorn, P.S. Anil Kumer, H. Ibach and J. Kirschner, Phys. Rev. Lett. **90**, 57001 (2003).

[37] H. Kotegawa, M. Yogi, Y. Imamura, Y. Kawasaki, G.-q. Zheng, Y. Kitaoka, S. Ohsaki, H. Sugawara, Y. Aoki, H. Sato, Phys. Rev. Lett. **90**, 27001 (2003).

[38] K. Maki, H. Won, P. Thalmeier, Q. Yuan, K. Izawa and Y. Matsuda, Europhys. Lett. **64**, 496 (2003).

[39] K. Aoki et al, Phys. Rev. Lett. **91**, 067003 (2003).

[40] K. Izawa et al (unpublished).

[41] H. Tou (private communication).

[42] E.E. Chia, M.B. Salomon, H. Sugawara, and H. Sato, Phys. Rev. Lett. **91**, 247003 (2003).

[43] H. Won, D. Parker, S. Haas and K. Maki, Current Appl. Phys. **4**, 523 (2004).

[44] H. Sugawara, S Osaki, S.R. Saha, Y. Aoki, H. Sato, Y. Inada, H. Shishido, R. Settai, Y. Onuki, H. Harima and K. Oikawa, Phys. Rev. B **66** 220504(R), (2002).

[45] C. Geibel et al, Z. Phys. B **84**, 1 (1991).

[46] H. Tou el at, J. Phys. Soc. Jpn. **64**, 725 (1995).

[47] J. Hessert et al, Physica B **230-232**, 373 (1997).

[48] May Chiao, B. Lussier, E. Elleman and L. Taillefer, Physica B **230-232**, 370 (1997).

[49] M. Jourdan, M. Huth and H. Adrian, Nature **398**, 47 (1999).

[50] P.A. Lee, Phys. Rev. Lett. **71**, 1887 (1993).

[51] Y. Sun and K. Maki, Europhys. Lett. **32**, 335 (1995).

[52] P. Thalmeier and K. Maki, Phys. Rev. B **67**, 92510 (2003).

[53] H. Tou, K. Ishida, Y. Kitaoka, cond-mat/0308562.